Critical exponents of the Ising model with competing Glauber and Kawasaki dynamics

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The two-dimensional Ising model with competing Glauber and Kawasaki dynamics is studied by Monte Carlo simulations. We show that the model exhibits the phenomenon of self-organization when the Kawasaki dynamics is the dominant one. For this model we show that the values of the critical exponents calculated at the stationary states are in accordance with the exact ones known for the equilibrium Ising model. These results give support to the idea that the equilibrium and nonequilibrium Ising models, which exhibit up-down symmetry, belong to the same universality class. [S1063-651X(96)06405-7]

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The study of nonequilibrium Ising spin systems appears naturally when the spins are subject to different dynamics. For instance, Gonzalez-Miranda *et al.* [1] have considered an Ising spin model where the Glauber [2] and the Kawasaki [3] dynamics drive the magnetic system. For the Glauber process, the system experiments single spin flips due to its contact with the heat bath at fixed temperature. On the other hand, for the Kawasaki process the transition rates are independent of spin configurations and this is equivalent to a contact with a heat bath of infinite temperature. When we consider only the independent configuration Kawasaki process, the correlation between sites are absent and the stationary states are of the Bernoulli type. As in the Kawasaki process the magnetization is conserved by the spin exchanges, the number of microscopic states is very large for a given value of the magnetization. Gonzalez-Miranda *et al.* [1] have found, for finite values of the Kawasaki exchange rate, the phase diagram of the model in the plane temperature versus the probability (p) of occurring the Kawasaki process. Their phase diagram, calculated through Monte Carlo simulations in two dimensions, shows a line of continuous transitions between the ferro- and paramagnetic phases. The temperature decreases as a function of p, and appears a nonequilibrium tricritical point, where the transition changes to first order. Dickman [4], using the pair approximation for the same model, also obtained an equivalent phase diagram with slightly different values for the location of the tricritical point. In particular, if the transition rate for the exchange of spins becomes very large, the fast exchange of spins leads to a diffusion-reaction equation for the local magnetization at infinite temperature [5,6].

Another very interesting feature about the competition between the Glauber and Kawasaki processes is the emergence of the phenomenon of self-organization [7]. This can be observed when the ferromagnetic Ising system is coupled to a heat bath, whose stochastic dynamics is given by the one-spin flip Glauber process, and subject to an external flux of energy, which can be simulated by a Kawasaki process that favors an increase in the energy of the system. It was shown in Ref. [7] that, within the dynamical pair approximation, and for a two-dimensional square lattice, the system goes continuously from the ferromagnetic to paramagnetic state as we increase the flux of energy. If we further increase the flux of energy, the system self-organizes into an antiferromagnetic phase. We would like to point out that the pair approximation gives no self-organization when the exchange coupling between the nearest-neighboring spins is of the antiferromagnetic type [8]. In this case, our two-dimensional calculations show that the antiferromagnetic order is destroyed by a small input of energy into the system.

We have wondered if Monte Carlo simulation on the two-dimensional version of the ferromagnetic system would maintain the picture of a self-organization phenomenon. As a matter of fact, we consider a ferromagnetic Ising model on a square lattice with N lattice sites. The state of the system is represented by $\sigma=(\sigma_1,\sigma_2,\ldots,\sigma_N)$, where the spin variable assumes the values $\sigma_i=\pm 1$. The energy of the system in the state σ is given by

$$E(\sigma)=-J\sum_{(i,\,j)}\sigma_i\sigma_j, \tag{1}$$

where in the summation only spins that are nearest neighbors are considered and $J>0$. Let $P(\sigma,t)$ be the probability of finding the system in the state σ at time t. The evolution of $P(\sigma,t)$ is given by the following master equation:

$$\frac{dP(\sigma,t)}{dt}=\sum_{\sigma'}\,[P(\sigma',t)W(\sigma',\sigma)-P(\sigma,t)W(\sigma,\sigma')], \tag{2}$$

where $W(\sigma,\sigma')$ gives the probability, per unit time, for the transition from the state σ' to state σ. We assume that the two competing processes can be written as

$$W(\sigma',\sigma)=pW_G(\sigma',\sigma)+(1-p)W_K(\sigma',\sigma). \tag{3}$$

In the above equation

$$W_G(\sigma',\sigma)=\sum_{i=1}^{N}\,\delta_{\sigma_1',\sigma_1}\delta_{\sigma_2',\sigma_2},\ldots,\delta_{\sigma_i',-\sigma_i},\ldots,\delta_{\sigma_N',\sigma_N}w_i(\sigma) \tag{4}$$

is the one-spin flip Glauber process which simulates the contact of our system with the heat bath at absolute temperature T, and

$$W_K(\sigma',\sigma) = \sum_{(i,\,j)} \delta_{\sigma_1',\sigma_1}\delta_{\sigma_2',\sigma_2},\ldots,\delta_{\sigma_i',\sigma_j},\ldots,\delta_{\sigma_j',\sigma_i},\ldots,\delta_{\sigma_N',\sigma_N} w_{ij}(\sigma) \tag{5}$$

is the two-spin exchange Kawasaki process, which simulates the flux of energy into the system. In these equations $w_i(\sigma)$ and $w_{ij}(\sigma)$ are, respectively, the probability, per unit time, of flipping spin i and the probability, per unit time, of exchanging two nearest-neighboring spins i and j. In the following we write the prescriptions we have taken for $w_i(\sigma)$ and $w_{ij}(\sigma)$:

$$w_i(\sigma) = \min\left[1,\exp\left(-\frac{\Delta E_i}{k_B T}\right)\right] \tag{6}$$

and

$$w_{ij}(\sigma) = \begin{cases} 0, & \text{for } \Delta E_{ij} \leqslant 0 \\ 1, & \text{for } \Delta E_{ij} > 0, \end{cases} \tag{7}$$

where ΔE_i is the change in energy after flipping spin i, and ΔE_{ij} is the change in energy after exchanging the neighboring spins i and j.

We have performed Monte Carlo simulations on a square lattice with $L\times L = N$ sites, with the values of L ranging from $L=6$ up to $L=80$. We have used in all of our simulations periodic boundary conditions. Also, we have started the simulations with different initial states in order to guarantee that the final stationary states we use in our calculations are the correct ones. For a given temperature T, and a chosen value of the probability p, we choose at random a spin i, from a given initial configuration. Then, we generate a random number ξ_1 between zero and unity. If $\xi_1 \leqslant p$ we choose to perform the Glauber process; in this process, we calculate the value of $w_i(\sigma)$. We again generate another random number ξ_2: if $\xi_2 \leqslant w_i(\sigma)$, we flip spin i, otherwise do not. If $\xi_1 > p$ we go over the Kawasaki process. We generate another random number ξ_3 in order to select one of the four nearest neighbors of the spin i, say j. Then we find the value of w_{ij} and we exchange the selected spins only if $w_{ij}=1$. We note that after $10^4\times N$ Monte Carlo steps the stationary regime was established, for all lattice sizes we consider. One Monte Carlo step equals N spin flip or exchange of spins trials. In order to estimate the quantities of interest, we have used 5×10^4 Monte Carlo steps to calculate the averages for any lattice size.

In order to locate the critical temperature for every value of p, we have plotted the reduced fourth-order cumulant [9] $U_L(T)$ [see Eq. (10) below] as a function of temperature T, for several values of L. The resulting phase diagram can be seen in Fig. 1, where we have plotted $\eta=\exp(-J/k_B T)$ as a function of $(1-p)$. Clearly, we can see that this phase diagram is rather different from that obtained through the dynamical pair approximation [7]. Here, we find a very small region in the phase diagram corresponding to the antiferromagnetic phase. Then, the self-organization, that is, the emergence of the antiferromagnetic phase from the disordered paramagnetic phase occurs only for high values of the flux of energy into the system. This phase occupies a narrow

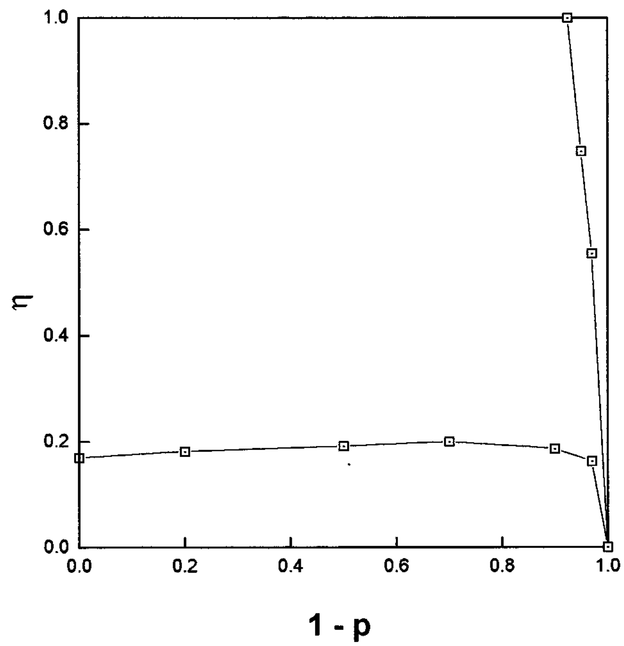

FIG. 1. Phase diagram of the two-dimensional kinetic ferromagnetic Ising model with competing Glauber (probability p) and Kawasaki (probability $1-p$) dynamics. The parameter η is given by $\eta=\exp(-J/k_B T)$. The system exhibits the paramagnetic (P), ferromagnetic (F), and antiferromagnetic (AF) phases. The broken lines serve as a guide to the eyes.

region of the phase diagram, with p between the values 0 and $p=0.075$. Differently of the work of Gonzalez-Miranda *et al.* [1] where the transition rate associated with the Kawasaki process was independent of the spin configurations, here we do not observe any dynamical tricritical behavior. On the other hand, the critical temperature exhibits a slight maximum around the value $p=0.3$. For values of $p<0.3$, where the Kawasaki process is the dominant one, the critical temperature for the stationary states decreases towards the zero temperature as $p\rightarrow 0$. For the pure Kawasaki case $p=0$ the evolution of the system is the same for whatever temperature, that is, we always go to a state of maximum energy compatible with a given initial magnetization. For instance, if the initial state is a paramagnetic one, the final state will be the one where the staggered magnetization per spin reaches its maximum value, that is, 1.

By employing the finite-size scaling relations [9,10] we can evaluate the stationary critical exponents associated with these transitions. Then for a system with $L\times L=N$ spins, with periodic boundary conditions, we define, at the stationary states, the ''magnetization'' per spin M_L and the ''susceptibility'' per spin χ_L as

$$M_L = \langle |m| \rangle, \tag{8}$$

$$\chi_L = \{\langle m^2\rangle - \langle |m|\rangle^2\}, \tag{9}$$

where $m = 1/N\Sigma_{i=1}^{N}\sigma_i$. We also define the reduced fourth-order cumulant U_L as

$$U_L = 1 - \frac{\langle m^4\rangle}{3\langle m^2\rangle^2}. \tag{10}$$

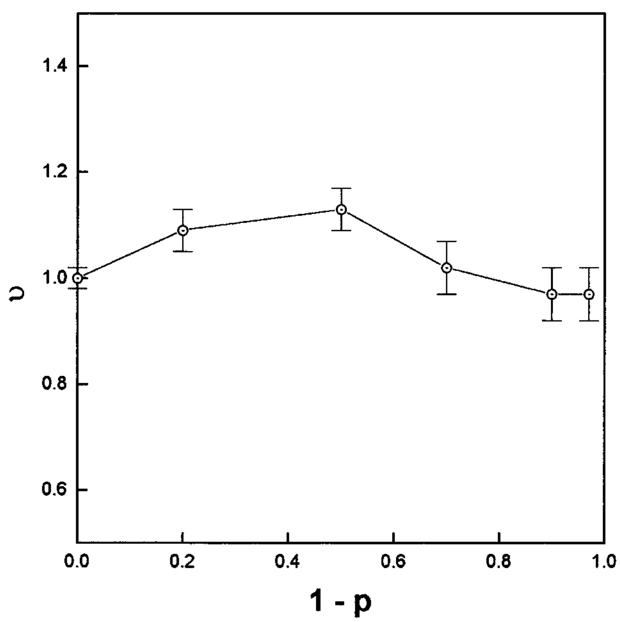

FIG. 2. Stationary correlation critical exponent ν as a function of $(1-p)$ at the transition point between the ferro- and paramagnetic phases. The error bars give the accuracy of our Monte Carlo data points. The estimated values of ν are around the corresponding equilibrium value $\nu=1$.

These above defined quantities obey the following finite-size scaling relations in the neighborhood of the stationary critical point:

$$M_L(T)=L^{-\beta/\nu}M_0(L^{1/\nu}\epsilon), \tag{11}$$

$$\chi_L(T)=L^{(\gamma/\nu)-2}\chi_0(L^{1/\nu}\epsilon), \tag{12}$$

$$U_L(T)=U_0(L^{1/\nu}\epsilon), \tag{13}$$

where $\epsilon=[(T-T_c)/T_c]$, T_c being the critical temperature for each value of p.

If we derive the Eq. (13) with respect the temperature T we obtain the following scaling relation:

$$U_L'(T)=L^{1/\nu}\frac{U_0'(L^{1/\nu}\epsilon)}{T_c}, \tag{14}$$

so that $U_L'(T_c)=L^{1/\nu}[U_0'(0)/T_c]$. Then, we can find the critical exponent ν from the slope of the straight line which is the best fit to the data point of $U_L'(T_c)$ for each value of L. In Fig. 2 we exhibit the behavior of ν as a function of $(1-p)$. Surprisingly, for this Ising model with competing Kawasaki and Glauber dynamics, the stationary critical exponent ν is almost equal to 1. This interesting behavior is in agreement with the arguments given by Grinstein, Jayaparash, and Yu He [11] that the equilibrium and nonequilibrium stochastic spin systems, which present the up-down symmetry, fall in the same universality class. In order to corroborate these arguments, we also calculate the stationary critical exponents β and γ from the log-log plots of the magnetization M_L and of the susceptibility χ_L as a function of L, respectively. For every log-log plot we have calculated the values of M_L and χ_L at the critical temperature $T_c(p)$, as

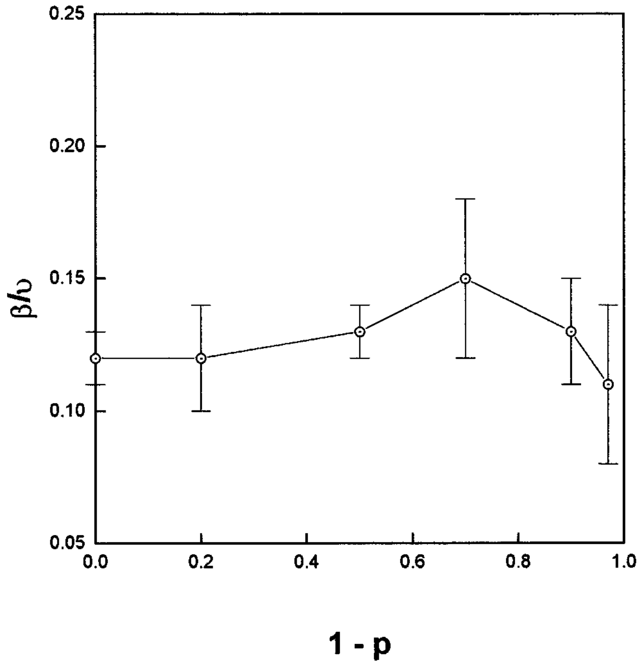

FIG. 3. Stationary values of the ratio β/ν as a function of $(1-p)$ at the transition point between the ferro- and paramagnetic phases. We see that our estimated values of this ratio oscillate around the exact equilibrium value 1/8.

exhibited in Fig. 1. In Figs. 3 and 4 we exhibit the results we have found for β/ν and γ/ν for several values of p, respectively. The exact values for the equilibrium Ising model exponents are well known and are given by $\nu=1$, $\beta=1/8$, and $\gamma=7/4$. As we can see our estimated values for β/ν and γ/ν,

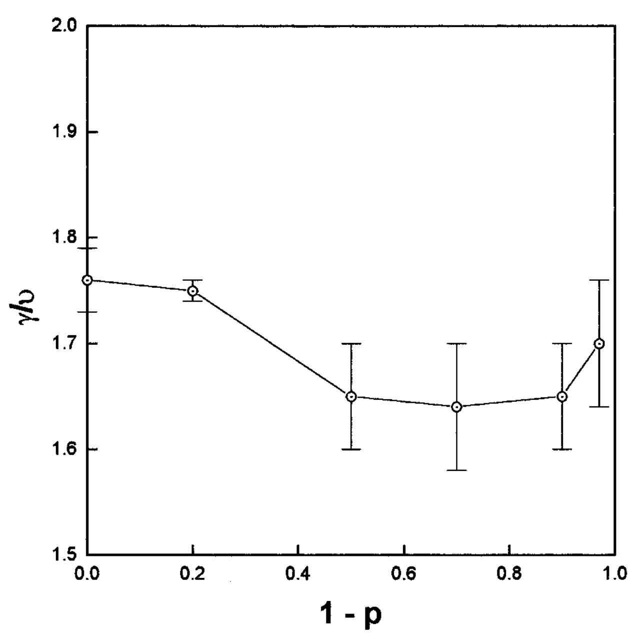

FIG. 4. Stationary values of the ratio γ/ν as a function of $(1-p)$ at the transition point between the ferro- and paramagnetic phases. Within the accuracy of our data points, the values of this ratio oscillate around the exact equilibrium value 7/4.

reported on Figs. 3 and 4, are in accordance with the corresponding values at equilibrium.

Although we do not present the detailed calculations concerning the continuous transition between the paramagnetic and antiferromagnetic phases, the critical temperature and the critical exponents can be obtained in a similar manner as we have done for the ferromagnetic-paramagnetic transition. For instance, if $p=0.03$, we have found the following values: $T_c=6.82\pm0.02$, $\nu=0.97\pm0.05$, $\beta/\nu=0.13\pm0.01$, and $\gamma/\nu=1.83\pm0.06$.

In conclusion, we have presented a very simple nonequilibrium model, where the stationary critical behavior is in the same universality class as the two-dimensional equilibrium Ising model. We think that our model, which exhibits the self-organization phenomenon, due the competition between the Glauber and configuration dependent Kawasaki dynamics, preserves the main features of the up-down symmetry. The stationary critical exponents we have found in our Monte Carlo simulations are in accordance with the exact ones known for the square Ising model with periodic boundary conditions.

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